practice

How would you find:

- $$\int \sqrt{x}\, e^{\sqrt{x}}\, \mathrm{d}x$$
- $$\int \frac{\mathrm{d}x}{\sqrt{x+1}+\sqrt{x}}$$
- $$\int \frac{\sqrt{x}}{1+\sqrt[3]{x}}\, \mathrm{d}x$$

1. Let u=sqrt(x). Integrate by parts twice.
   Answer=2 (x-2sqrt(x)+2) e^x + C
   (Stewart 7.5.63)

2. Multiply and divide by conjugate, denominator becomes 1.
   Answer=2/3*((x+1)^(3/2)-x^(3/2))+C
   (Stewart 7.5.65)

3. Let u=x^(1/6). Do long division.
   Answer=6 (1/7*x^(7/6) - 1/5 * x^(5/6)
   + 1/3 * x^(1/2) - x^(1/6)
   + arctan(x^(1/6)) + C
   (Stewart 7.5.72)

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today:
§ 8.1 - arc length

thursday:
§ 8.2 - surface area
quiz iv: §§ 4.4, 7.4
homework 6 due (4.4.28, 4.4.40, 4.4.58, 7.8.26, 7.8.36, 7.8.40)

monday:
webwork extra credit ii help session in EA 265 @ 5:30

tuesday, 17 november:
homework 7 due (8.1.6, 8.1.16, 8.1.34, 8.2.12, 8.2.14, 8.2.26)
review for midterm iii

thursday, 19 november:
midterm iii: §§ 4.4, 7.4, 7.8, 8.1, 8.2
§ 9.1 - modeling with differential equations

monday, 23 november:
webwork extra credit ii due @ 6:00 am

Correction from last Thursday's slides: quiz iv is on §§ 4.4 and 7.4, NOT 7.8.

improper integrals

evaluate

$$\int_0^\infty x\, e^{-x}\, \mathrm{d}x$$

This is an improper integral of type 1.

Answer=1

(Integration by parts)

improper integrals

evaluate

$$\int_0^3 \frac{\mathrm{d}x}{\sqrt{x}}$$

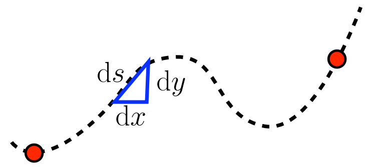
This is an improper integral of type 2.

Answer=2 sqrt(3)

(Discontinuous at 0.)
Stewart 7.8.27.

arc length

Suppose I want to find the length of the portion of the dashed curve between the two red dots.



To do this, we chop up the length into infinitely many infinitely small segments, each one of which is a line segment with varying lengths $\mathrm{d}s$.

$$L = \int_?^? \mathrm{d}s$$

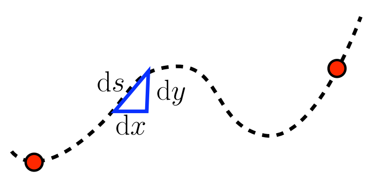
The length L is the integral of the lengths of the line segments ds.

I'm not worrying about bounds yet.

Bigger question: what is ds?

arc length

But $\mathrm{d}s$ is the hypotenuse of a triangle with infinitesimal width $\mathrm{d}x$ and infinitesimal height $\mathrm{d}y$.



Thus

$$\begin{aligned} \mathrm{d}s &= \sqrt{\mathrm{d}x^2 + \mathrm{d}y^2} \\ &= \sqrt{1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\, \mathrm{d}x \\ &= \sqrt{1 + \left(\frac{\mathrm{d}x}{\mathrm{d}y}\right)^2}\, \mathrm{d}y \end{aligned}$$

The second line follows by factoring out a dx^2, the third by factorying out a dy^2 instead.

arc length

Suppose f ' is continuous on $[a, b]$. Then the length of $y=\mathrm{f}(x)$, $a \leq x \leq b$ is

$$L = \int_a^b \sqrt{1 + (f'(x))^2}\, \mathrm{d}x$$

arc length

Suppose g ' is continuous on $[c, d]$. Then the length of $x=\mathrm{g}(y)$, $c \leq y \leq d$ is

$$L = \int_c^d \sqrt{1 + (g'(y))^2}\, \mathrm{d}y$$

This follows immediately from the first form by changing variables... it also follows from the last expression for ds since g'(y)=dx/dy

example

find the length of the following, for $0 \leq x \leq 1$

$$y = \sqrt{1 - x^2}$$

y'=-x/sqrt(1-x^2)

Use a trig sub, x=sin(theta)

L=pi/2

(This is the length of the portion of the unit circle in the first quadrant... D=2pi, so L=D/4, so L=pi/2)

example

find the length of the following, for $1 \leq y \leq 2$

$$x = \frac{y^5}{6} + \frac{1}{10\, y^3}$$

1+(dx/dy)^2=(5/6 x^4 + 3/10 x^(-4))^2

Answer=31/6+7/80=1261/240

Essentially Stewart 8.1.7.

example

find the length of the following, for $1 \leq x \leq 4$

$$y = \int_1^x \sqrt{t^3 - 1}\,\mathrm{d}t$$

dy/dx by the fundamental theorem of calculus,

answer=62/5

Stewart 8.1.37

arc length function

Suppose $f\,'$ is continuous on $[a,\ b]$. Then the function

$$s(x) = \int_a^x \sqrt{1 + (f'(t))^2}\,\mathrm{d}t$$

is called the arc length function from the point $P_0 = (a,\ f(a))$.

example

find the arc length function of $y = 2\ x^{3/2}$ with starting point $(1,\ 7/12)$.

Use your answer to find the length along the curve from $(1,\ 7/12)$ to $(4,\ 16)$...

...and from $(4,\ 16)$ to $(9,\ 54)$.

Integrate from 1 to x to find s(x).

Answer:
(a) s(x)=2/27*((1+9x)^(3/2) - 10^(3/2)) <-- Stewart 8.1.31
(b) 2/27*((1+9*4)^(3/2) - 10^(3/2))
(c) 2/27*((1+9*9)^(3/2) - 10^(3/2)) - 2/27*((1+9*4)^(3/2) - 10^(3/2))

For (c), note that the distance between the points is the distance from 1 to 9 minus the distance from 4 to 9. Same as with cumulative area functions.

coming soon

- read § 8.2
- quiz iv (§§ 4.4, 7.4) on thursday
- homework 6 due thursday
- start extra credit project 2, due 23 november @ 6:00 am